Osobine Laplasove transformacije

1. Translacija po s

Ako Laplasova transformacija $\mathcal{L}\{f\}(s) = F(s)$ postoji za $s > \alpha$ tada

$$\mathcal{L}\{e^{at} f(t)\}(s) = F(s-a)$$

za $s > \alpha + a$.

2. Laplasova transformacija izvoda

Neka je $f(t)$ neprekidna na $[0, \infty)$, i neka je $f'(t)$ po dijelovima neprekidna na $[0, \infty)$, gdje su obe f-je eksponencijalne reda α. Tada, za $s > \alpha$

$$\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$$

3. Laplasova transformacija izvoda višeg reda

Neka su $f(t), f'(t), \dots, f^{(n-1)}(t)$ neprekidne f-je na $[0, \infty)$ i neka je $f^{(n)}(t)$ po dijelovima neprekidna na $[0, \infty)$ gdje su sve ove f-je eksponencijalne reda α. Tada, za $s > \alpha$

$$\mathcal{L}\{f^{(n)}\}(s) = s^n \mathcal{L}\{f\}(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0).$$

4. Izvod Laplasove transformacije

Neka je $F(s) = \mathcal{L}\{f\}(s)$ i pretpostavimo da je $f(t)$ po djelovima neprekidna na $[0, \infty)$ i eksponencijalna reda α. Tada za $s > \alpha$

$$\mathcal{L}\{t^n f(t)\}(s) = (-1)^n \frac{d^n F}{ds^n}(s).$$

5. Integriranje slike

Neka je $F(s)=\mathcal{L}\{f\}(s)$. Tada $\mathcal{L}\left\{\frac{f(t)}{t}\right\}(s)=\int_s^\infty F(x)\,dx$.

6. Integriranje orginala

Ako je $F(s)=\mathcal{L}\{f\}(s)$ tada $\mathcal{L}\left\{\int_0^t f(x)\,dx\right\}(s)=\frac{F(s)}{s}$

Prema tome osnovne osobine Laplasove transformacije su

$$\mathcal{L}\{f+g\}=\mathcal{L}\{f\}+\mathcal{L}\{g\}.$$

$\mathcal{L}\{cf\}=c\,\mathcal{L}\{f\}$ za bilo koju konstantu c

$$\mathcal{L}\{e^{at}f(t)\}(s)=\mathcal{L}\{f\}(s-a)$$

$$\mathcal{L}\{f'\}(s)=s\,\mathcal{L}\{f\}(s)-f(0)$$

$$\mathcal{L}\{f''\}(s)=s^2\mathcal{L}\{f\}(s)-sf(0)-f'(0)$$

$$\mathcal{L}\{f^{(n)}\}(s)=s^n\mathcal{L}\{f\}(s)-s^{n-1}f(0)-s^{n-2}f'(0)-\dots-f^{(n-1)}(0)$$

$$\mathcal{L}\{t^n f(t)\}(s)=(-1)^n\frac{d^n}{ds^n}\left(\mathcal{L}\{f\}(s)\right).$$

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\}(s)=\int_s^\infty F(x)\,dx$$

$$\mathcal{L}\left\{\int_0^t f(x)\,dx\right\}(s)=\frac{F(s)}{s}$$

(#) Odrediti Laplasovu transformaciju f-je $e^{\alpha t}\sin\beta t$.

Rj: U rješenju zadatka ćemo iskoristiti osobinu translacije po s

$$\underline{\mathcal{L}\{e^{at}f(t)\}(s)=F(s-a)}$$

Od ranije znamo $\mathcal{L}\{\sin\beta t\}(s)=\frac{\beta}{s^2+\beta^2}$

tj. $F(s)=\frac{\beta}{s^2+\beta^2}$

Sad ako iskoristimo osobinu translacije

$$\mathcal{L}\{e^{\alpha t}\sin\beta t\}(s)=F(s-\alpha)=\frac{\beta}{(s-\alpha)^2+\beta^2}.$$

(#) Izračunati $\mathcal{L}\{t^n e^{2t}\}(s)$.

Rj. Znamo da ako je $\mathcal{L}\{f(t)\}(s) = F(s)$ tada

$$\mathcal{L}\{e^{at} f(t)\}(s) = F(s-a)$$

S obzirom da je

$$\mathcal{L}\{t^n\}(s) = \frac{n!}{s^{n+1}}, \quad s > 0$$

(elementarna tabela Laplasovih transformacija)

to je $\mathcal{L}\{e^{2t} t^n\} = \dfrac{n!}{(s-2)^{n+1}}$

(#) Koristeći Laplasovu transformaciju izvoda $\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$ odrediti $\mathcal{L}\{\sin\omega t\}(s)$.

Rj.

Neka je $\mathcal{L}\{\sin\omega t\}(s) = F(s)$.

$$(\sin\omega t)' = \omega\cos\omega t$$

$$\mathcal{L}\{(\sin\omega t)'\}(s) = s\mathcal{L}\{\sin\omega t\}(s) - \sin\omega 0 = sF(s)$$

$$\parallel$$

$$\mathcal{L}\{\omega\cos\omega t\}(s) = \omega\mathcal{L}\{\cos\omega t\}(s)$$

tj. $$\mathcal{L}\{\cos\omega t\}(s) = \frac{s}{\omega}F(s)$$

$$(\cos\omega t)' = -\omega\sin\omega t$$

$$\mathcal{L}\{(\cos\omega t)'\}(s) = s\cdot\frac{s}{\omega}F(s) - \cos\omega 0 = \frac{s^2}{\omega}F(s) - 1$$

$$\parallel$$

$$\mathcal{L}\{-\omega\sin\omega t\}(s) = -\omega\mathcal{L}\{\sin\omega t\}(s) = -\omega F(s)$$

tj. $$-\omega F(s) = \frac{s^2}{\omega}F(s) - 1$$

$$\frac{s^2}{\omega}F(s) + \omega F(s) = 1$$

$$\frac{s^2+\omega^2}{\omega}F(s) = 1 \quad\Rightarrow\quad F(s) = \frac{\omega}{s^2+\omega^2}$$

tj. $$\mathcal{L}\{\sin\omega t\}(s) = \frac{\omega}{s^2+\omega^2}$$

(#) Koristeći Laplasovu transformaciju izvoda $\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$ odrediti $\mathcal{L}\{\sin t\}(s)$.

Rj.

Neka je $\mathcal{L}\{\sin t\}(s) = F(s)$.

$$(\sin t)' = \cos t$$

$$\mathcal{L}\{(\sin t)'\}(s) = s\mathcal{L}\{\sin t\}(s) - \sin(0)$$
$$= sF(s)$$

tj. $\mathcal{L}\{\cos t\} = sF(s)$

$$(\cos t)' = -\sin t$$

$$\mathcal{L}\{(\cos t)'\}(s) = s\mathcal{L}\{\cos t\}(s) - \cos 0$$
$$= s^2 F(s) - 1$$

tj.

$$\mathcal{L}\{-\sin t\} = s^2 F(s) - 1$$

Kako je $\mathcal{L}\{-\sin t\} = -\mathcal{L}\{\sin t\} = -F(s)$

to je

$$-F(s) = s^2 F(s) - 1$$

$$(s^2+1)F(s) = 1 \Rightarrow F(s) = \frac{1}{s^2+1}$$

tj.

$$\mathcal{L}\{\sin t\}(s) = \frac{1}{s^2+1}$$

(#) Koristeći osobine Laplasove transformacije i činjenicu da je $\mathcal{L}\{\sin bt\}(s) = \frac{b}{s^2+b^2}$ odrediti $\mathcal{L}\{\cos bt\}$.

Rj: U rješenju ćemo iskoristiti osobinu Laplasove transformacije izvoda

$$\underline{\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)}$$

Neka je $f(t) := \sin bt$. Tada $f(0) = 0$

$$f'(t) = b\cos bt$$

Uvrstimo ovo u napisanu formulu

$$\mathcal{L}\{b\cos bt\}(s) = s\mathcal{L}\{\sin bt\}(s) - 0$$

$$b\mathcal{L}\{\cos bt\}(s) = \frac{sb}{s^2+b^2} \qquad /:b$$

$$\mathcal{L}\{\cos bt\}(s) = \frac{s}{s^2+b^2}$$

(#) Pokazati da za svaku neprekidnu f-ju $f(t)$ vrijedi sljedeća jednakost

$$\mathcal{L}\left\{\int_0^t f(x)\,dx\right\}(s) = \frac{1}{s}\mathcal{L}\{f(t)\}(s)$$

(predpostavljajući da transformacija postoji).

Rj: Definišimo f-ju $g(t)$ na sljedeći način $g(t) := \int_0^t f(x)\,dx$.

Primjetimo da je $g(0) = 0$ i $g'(t) = f(t)$.

Sad iskoristimo osobinu Laplasove transformacije izvoda

$$\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$$

(umjesto $f(t)$ mi imamo $g(t)$)

$$\mathcal{L}\{f(t)\}(s) = s\mathcal{L}\left\{\int_0^t f(x)\,dx\right\}(s) - 0$$

iz čega slijedi

$$\mathcal{L}\left\{\int_0^t f(x)\,dx\right\}(s) = \frac{1}{s}\mathcal{L}\{f(t)\}(s)$$

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(#) Laplasova transformacija f-je $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

(koja se naziva step f-ja ili jedinična f-ja) je $\mathcal{L}\{u(t)\}(s) = \frac{1}{s}$ gdje $s > 0$. Koristeći ovu činjenicu i osobinu izvoda Laplasove transformacije $\mathcal{L}\{t^n f(t)\}(s) = (-1)^n \frac{d^n F}{ds^n}(s)$ odrediti $\mathcal{L}\{t^n\}$.

Rj.

$$\mathcal{L}\{u(t)\}(s) = \frac{1}{s} = F(s)$$

$$\mathcal{L}\{-t\,u(t)\}(s) = -\mathcal{L}\{t\,u(t)\} = -(-1)^1 \frac{dF}{ds}(s)$$

$$= \frac{d}{ds}\left(\frac{1}{s}\right) = (-1)\, s^{-2} = \frac{-1}{s^2}$$

$$\Rightarrow \quad \mathcal{L}\{-t\}(s) = \frac{-1}{s^2} \quad \Rightarrow \quad \mathcal{L}\{t\}(s) = \frac{1}{s^2}$$

$$\mathcal{L}\{(-t)^2 u(t)\}(s) = \mathcal{L}\{t^2 u(t)\} = (-1)^2 \frac{d^2}{ds^2} F(s) = \frac{\overset{(-1)(-2)}{2}}{s^3}$$

$$\Rightarrow \quad \mathcal{L}\{t^2\}(s) = \frac{2}{s^3}$$

$$\mathcal{L}\{(-t)^n u(t)\}(s) = (-1)^n \mathcal{L}\{t^n u(t)\}(s) = (-1)^n (-1)^n \frac{d^n}{ds^n} F(s)$$

$$= \frac{d^n}{ds^n}\left(\frac{1}{s}\right) = \frac{(-1)(-2)\dots(-n)}{s^{n+1}} = \frac{(-1)^n n!}{s^{n+1}}$$

tj. $$(-1)^n \mathcal{L}\{t^n\}(s) = (-1)^n \frac{n!}{s^{n+1}} \quad \Rightarrow \quad \mathcal{L}\{t^n\}(s) = \frac{n!}{s^{n+1}}$$

(#) Odrediti $\mathcal{L}\{t \sin bt\}$.

Rj. Iz tablice elementarnih transformacija znamo da je

$$\mathcal{L}\{\sin bt\}(s) = F(s) = \frac{b}{s^2+b^2}$$

Sad ako iskoristimo osobinu izvoda Laplasove transform.

$$\underline{\mathcal{L}\{t^n f(t)\}(s) = (-1)^n \frac{d^n F}{ds^n}(s)} \qquad \ldots (*)$$

diferencirajući $F(s)$ dobićemo

$$\frac{dF}{ds}(s) = \frac{0 - b \cdot 2s}{(s^2+b^2)} = \frac{-2bs}{(s^2+b^2)}$$

Koristeći $(*)$ imamo

$$\mathcal{L}\{t \sin bt\}(s) = -\frac{dF}{ds}(s) = \frac{2bs}{(s^2+b^2)^2}.$$

(#) Odrediti $\mathcal{L}\{te^t\cos t\}$.

Rj. Iz tablice elementarnih transformacija

$$\mathcal{L}\{\cos t\}(s) = \frac{s}{s^2+1} \quad \text{gdje} \quad s>0.$$

Osobina translacije po s kaže da za $\mathcal{L}\{f\}(s) = F(s)$

$$\underline{\mathcal{L}\{e^{at}f(t)\}(s) = F(s-a)}$$

U našem slučaju

$$\mathcal{L}\{e^t\cos t\}(s) = \frac{s-1}{(s-1)^2+1} = \frac{s-1}{s^2-2s+2}$$

Za $F(s) = \mathcal{L}\{f\}(s)$ osobina izvoda Laplasove transformacije je

$$\underline{\mathcal{L}\{t^n f(t)\}(s) = (-1)^n \frac{d^n}{ds^n}F(s)}$$

pa imamo

$$\mathcal{L}\{te^t\cos t\}(s) = (-1)^1 \frac{d}{ds}\left(\frac{s-1}{s^2-2s+2}\right)$$

$$= (-1)\frac{1\cdot(s^2-2s+2) - \overbrace{(s-1)(2s-2)}^{2s^2-4s+2}}{(s^2-2s+2)^2} = (-1)\frac{-s^2+2s}{(s^2-2s+2)^2}$$

$$\mathcal{L}\{te^t\cos t\}(s) = \frac{s^2-2s}{(s^2-2s+2)^2}$$

(#) Odrediti sliku f-je $t f''(t)$ (drugim riječima odrediti $\mathcal{L}\{t f''(t)\}$.

Rj.

Znamo da (Laplasova transformacija izvoda)

$$\mathcal{L}\{f''\}(s) = s^2 \mathcal{L}\{f\}(s) - s f(0) - f'(0)$$

Ako označimo $F(s) = \mathcal{L}\{f\}(s)$

$$\mathcal{L}\{f''\}(s) = s^2 F(s) - s f(0) - f'(0)$$

Isto tako znamo (izvod Laplasove transformacije)

$$\mathcal{L}\{t f(t)\}(s) = (-1)^1 \frac{d}{ds} F(s)$$

Pa je

$$\mathcal{L}\{t f''(t)\} = (-1)\left[2 s F(s) + s^2 F'(s) - f(0)\right]$$

$$= -s^2 F'(s) - 2 s F(s) + f(0)$$

Prema tome

$$\mathcal{L}\{t f''(t)\}(s) = -s^2 F'(s) - 2 s F(s) + f(0)$$

(#) Odrediti Laplasovu transformaciju f-je $\dfrac{e^{-3t}-e^{-5t}}{t}$.

Rj. Prema tablici elementarnih transformacija imamo

$$\mathcal{L}\{e^{-3t}\}(s)=\frac{1}{s+3} \quad i \quad \mathcal{L}\{e^{-5t}\}(s)=\frac{1}{s+5}$$

pa vrijedi

$$\mathcal{L}\{e^{-3t}-e^{-5t}\}(s)=\frac{1}{s+3}-\frac{1}{s+5}$$

Ako sad primjenimo osobinu integriranja Laplasove transform.

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\}(s)=\int_s^\infty F(x)\,dx$$

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imamo

$$\mathcal{L}\left\{\frac{e^{-3t}-e^{-5t}}{t}\right\}(s)=\int_s^\infty\left(\frac{1}{x+3}-\frac{1}{x+5}\right)dx$$

$$=\left(\ln(x+3)-\ln(x+5)\right)\Big|_s^\infty=\ln\frac{x+3}{x+5}\Big|_s^\infty=$$

$$=\ln 1-\ln\frac{s+3}{s+5}=\ln\frac{s+5}{s+3}.$$

$$\mathcal{L}\left\{\frac{e^{-3t}-e^{-5t}}{t}\right\}(s)=\ln\frac{s+5}{s+3}$$

Koristeći Laplaceovu transformaciju, izračunati integral

$$\int_0^\infty \frac{e^{-t}\sin^2 t}{t}\,dt .$$

Rj.

Prema osobinama Laplaceove transformacije (integriranje slike) znamo da

$$F(s) = \mathcal{L}\{f\}(s) \implies \mathcal{L}\left\{\frac{f(t)}{t}\right\}(s) = \int_s^\infty F(x)\,dx \quad \ldots(*)$$

Prema definiciji Laplaceove transformacije

$$\mathcal{L}\left\{\frac{\sin^2 t}{t}\right\}(s) = \int_0^\infty e^{-st}\,\frac{\sin^2 t}{t}\,dt = F(s)$$

Prema tome dati integral je jednak $F(1)$ gdje je $F(s)$ slika f-je $\frac{\sin^2 t}{t}$ tj. $F(s) = \mathcal{L}\left\{\frac{\sin^2 t}{t}\right\}(s)$.

$$\mathcal{L}\{\sin^2 t\}(s) = \mathcal{L}\left\{\frac{1}{2}(1-\cos 2t)\right\}(s) = \frac{1}{2}\mathcal{L}\{1-\cos 2t\}(s) =$$

$$= \frac{1}{2}\mathcal{L}\{1\} - \frac{1}{2}\mathcal{L}\{\cos 2t\} = \frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2+4}\right)$$

$$\mathcal{L}\left\{\frac{\sin^2 t}{t}\right\}(s) \overset{(*)}{=} \frac{1}{2}\int_s^\infty \left(\frac{1}{x} - \frac{x}{x^2+4}\right)dx = \frac{1}{2}\Big[\underbrace{\ln x}_{\frac{1}{2}\ln x^2} - \frac{1}{2}\ln(x^2+4)\Big]\Big|_s^\infty$$

$$= \frac{1}{4}\ln\frac{x^2}{x^2+4}\Big|_s^\infty = \frac{1}{4}\ln\frac{s^2}{s^2+4} = F(s)$$

Prema tome $\displaystyle\int_0^\infty \frac{e^{-t}\sin^2 t}{t}\,dt = F(1) = \frac{1}{4}\ln 5$

(#) Izračunati integral $\int_0^\infty e^{-\sqrt{2}t}\frac{\operatorname{sh}t\sin t}{t}\,dt$

Rj Prema definiciji Laplaceove transformacije

$$\mathcal{L}\left\{\frac{\operatorname{sh}t\sin t}{t}\right\}(s)=\int_0^\infty e^{-st}\frac{\operatorname{sh}t\sin t}{t}\,dt=F(s)$$

Prema tome dati integral je jednak vrijednosti $F(\sqrt{2})$.

Prema tabeli elementarnih transformacija

$$\mathcal{L}\{\sin\beta t\}(s)=\frac{\beta}{s^2+\beta^2}\,,\qquad s>0$$

$$\mathcal{L}\{e^{\alpha t}\sin\beta t\}(s)=\frac{\beta}{(s-\alpha)^2+\beta^2}\,,\qquad s>\alpha$$

pa je

$$\mathcal{L}\{\operatorname{sh}t\sin t\}(s)=\frac{1}{2}\mathcal{L}\{(e^{t}-e^{-t})\sin t\}(s)=$$

$$=\frac{1}{2}\left[\frac{1}{(s-1)^2+1}-\frac{1}{(s+1)^2+1}\right]$$

Prema osobini integriranje slike

$$F(s)=\mathcal{L}\{f\}(s)\quad\Rightarrow\quad\mathcal{L}\left\{\frac{f(t)}{t}\right\}(s)=\int_s^\infty F(x)\,dx$$

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imamo

$$\mathcal{L}\left\{\frac{\operatorname{sh}t\sin t}{t}\right\}(s)=\frac{1}{2}\int_s^\infty\left[\frac{1}{(x-1)^2+1}-\frac{1}{(x+1)^2+1}\right]dx=$$

$$= \frac{1}{2}\left[\operatorname{arctg}(x-1)\Big|_s^\infty - \operatorname{arctg}(x+1)\Big|_s^\infty\right] =$$

$$= \frac{1}{2}\left(\frac{\pi}{2} - \operatorname{arctg}(s-1) - \frac{\pi}{2} + \operatorname{arctg}(s+1)\right)$$

$$= \frac{1}{2}\left(\operatorname{arctg}(s+1) - \operatorname{arctg}(s-1)\right) = F(s)$$

Prema tome traženi integral je jednak

$$\int_0^\infty e^{-\sqrt{2}t}\,\frac{\operatorname{sh} t \sin t}{t}\,dt = F(\sqrt{2}) = \frac{1}{2}\left(\operatorname{arctg}(\sqrt{2}+1) - \operatorname{arctg}(\sqrt{2}-1)\right)$$

Vrijednost dobivenog integrala se može dovesti na prikladniji oblik korištenjem formule

$$\boxed{\operatorname{arctg} a - \operatorname{arctg} b = \operatorname{arctg}\frac{a-b}{1+ab}}$$

Odavde

$$F(\sqrt{2}) = \frac{1}{2}\operatorname{arctg}\frac{(\sqrt{2}+1)-(\sqrt{2}-1)}{1+(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{1}{2}\operatorname{arctg}\underbrace{\frac{2}{2}}_{=1} = \frac{\pi}{8}$$

Odrediti Laplasovu transformaciju f-je

$$\varphi(t) = \int_0^t \frac{\sin u}{u}\, du$$

Rj.

$$\mathcal{L}\{\sin u\}(s) = \frac{1}{s^2+1}$$

Iz osobine Laplasove transformacije (integriranje slike)

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\}(s) = \int_s^\infty F(x)\, dx$$

imam

$$\mathcal{L}\left\{\frac{\sin t}{t}\right\}(s) = \int_s^\infty \frac{1}{x^2+1}\, dx = \operatorname{arc\,tg} x \Big|_s^\infty = \frac{\pi}{2} - \operatorname{arc\,tg} s$$

Prema osobini integriranja orginala

$$\mathcal{L}\left\{\int_0^t f(x)\, dx\right\}(s) = \frac{F(s)}{s}$$

imamo

$$\mathcal{L}\left\{\int_0^t \frac{\sin u}{u}\, du\right\}(s) = \frac{\frac{\pi}{2} - \operatorname{arc\,tg} s}{s}$$

Odrediti Laplasovu transformaciju f-je

$$\psi(t)=\int_0^t (e^{-3x}\operatorname{ch}2x+e^{4x}\sin 2x)\,dx$$

Rj: Iz tabele elementarnih Laplasovih transformacija znamo

$$\mathcal{L}\{\operatorname{ch}ax\}(s)=\frac{s}{s^2-a^2},\quad s>|a| \quad i \quad \mathcal{L}\{\sin ax\}(s)=\frac{a}{s^2+a^2},\quad s>0$$

pa je

$$\mathcal{L}\{\operatorname{ch}2x\}(s)=\frac{s}{s^2-4} \quad i \quad \mathcal{L}\{\sin 2x\}(s)=\frac{2}{s^2+4}$$

Iz osobine translacije po s

$$\mathcal{L}\{e^{at}f(t)\}(s)=F(s-a)$$

pa je

$$\mathcal{L}\{e^{-3x}\operatorname{ch}2x\}(s)=\frac{s+3}{(s+3)^2-4}=\frac{s+3}{s^2+6s+5}$$

$$\mathcal{L}\{e^{4x}\sin 2x\}(s)=\frac{2}{(s-4)^2+4}=\frac{2}{s^2-8s+20}$$

Na kraju, prema osobini integrisanju orginala

$$\mathcal{L}\left\{\int_0^t f(x)\,dx\right\}(s)=\frac{F(s)}{s}$$

$$\mathcal{L}\left\{\int_0^t (e^{-3x}\operatorname{ch}2x+e^{4x}\sin 2x)\,dx\right\}(s)=\frac{\frac{s+3}{s^2+6s+5}+\frac{2}{s^2-8s+20}}{s}$$

$$=\frac{s+3}{s^3+6s^2+5s}+\frac{2}{s^3-8s^2+20s}$$

Zadaci za vježbu

1. Odrediti

(a) $\mathcal{L}\{\operatorname{sh}2t\cdot\cos 2t\}(s)$ (b) $\mathcal{L}\{\operatorname{ch}2t\cdot\cos 2t\}(s)$

(c) $\mathcal{L}\{\operatorname{sh}^3(2t)\}(s)$

(d) $\mathcal{L}\{(t+1)\sin 2t\}(s)$ (e) $\mathcal{L}\{(t+1)^3 e^{-2t}\}(s)$

2. Odrediti

(a) $\mathcal{L}\left\{\dfrac{\cos 2t-\cos 3t}{t}\right\}(s)$; (b) $\mathcal{L}\left\{\dfrac{\operatorname{sh}t}{t}\right\}(s)$;

(c) $\mathcal{L}\left\{\dfrac{\sin 3t}{t}\right\}(s)$; (d) $\mathcal{L}\left\{\displaystyle\int_0^t \frac{\operatorname{sh}^2 u}{u}\,du\right\}(s)$

3. Primjenom Laplaceove transformacije izračunati integral

(a) $\displaystyle\int_0^\infty e^{-ax}\frac{\sin x}{x}\,dx$; (b) $\displaystyle\int_0^\infty e^{-ax}\frac{\sin^2 x}{x}\,dx$; (c) $\displaystyle\int_0^\infty e^{-2t}\,t\cos t\,dt$

(d) $\displaystyle\int_0^\infty e^{-3t}\,t\sin t\,dt$; (e) $\displaystyle\int_0^\infty \frac{\sin^2 t}{t^2}\,dt$; (f) $\displaystyle\int_0^\infty \frac{e^{-2t}-e^{-4t}}{t}\,dt$

Odgovori

1. (a) $\frac{2s^2-16}{s^4+64}$ (b) $\frac{s^3}{s^4+64}$ (c) $\frac{48}{(s^2-36)(s^2-4)}$

2. (a) $\frac{1}{2}\ln\frac{s^2+9}{s^2+4}$ (b) $\frac{1}{2}\ln\frac{s+1}{s-1}$

(c) $\frac{\pi}{2}-\operatorname{arctg}\frac{s}{3}$ (d) $\frac{1}{4s}\ln\left(\frac{s^2}{s^2-4}\right)$

3. (a) $\frac{\pi}{2}-\operatorname{arctg} a$ (b) $\frac{1}{2}\ln\frac{\sqrt{a^2+4}}{a}$

(c) $\frac{3}{25}$ (d) $\frac{3}{50}$ (e) $\frac{\pi}{2}$

4. Odrediti Laplasove transformacije sljedećih f-ja

(a) $f(t)=\int_0^t e^{-2u}\sin 3u\,du$ (b) $\int_0^t u\sin u\,du$

(c) $\varphi(t)=\int_0^t u^2e^u\,du$ (d) $\psi(t)=\int_0^t (u^3+1)e^{-u}\,du$

U PHOTOSHOPU 4. ti zadatak UBACI PRIJE ODGOV.